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Decomposability and the social comparison trap ☆

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ABSTRACT

We propose a unified and tractable model for social comparison preferences that nests competitive motives and imperfect inequality aversion as special cases. We identify a key property called decomposability and show that players are strategically independent in decomposable games under social comparison preferences, yielding a generically unique solution in every decomposable game. The class of decomposable games is broad, including every 2×2 symmetric game as well as multi-player, multi-strategy games such as the minimum effort game, public goods game, and tragedy of the commons. Using this characterization, we show that social comparison often results in undesirable outcomes, sometimes worse than any outcome that would arise if all players were material payoff maximizers.

1. Introduction

> In any community where goods are held in severalty it is necessary, in order to his own peace of mind, that an individual should possess as large a portion of goods as others with whom he is accustomed to class himself; and it is extremely gratifying to possess something more than others.
>
> [Thorstein Veblen, *The Theory of the Leisure Class* (1899).]

Social comparison is a central phenomenon in human societies. Since Festinger (1954), extensive evidence shows that individuals assess their abilities, status, and satisfaction by comparing themselves to others (Card et al., 2012; Luttmer, 2005; Clark and Oswald, 1996; Fliessbach et al., 2007; Wood, 1989). This process affects various economic behaviors, from consumption and saving decisions to workplace effort and public goods contributions (Frank, 1987; Cres et al., 1997; Mas, 2006; Rayo and Becker, 2007; Hopkins and Kornienko, 2004; Arrow and Dasgupta, 2009). Despite its recognized role in decision-making, social comparison is underexplored in game-theoretic analyses. Moreover, although competitive motives, or the uncertainty about opponents' such motives, may drive detrimental antisocial and destructive behaviors in various settings (Zizzo and Oswald, 2001; Herrmann et al., 2008; Abbink and Sadrieh, 2009; Abbink and Herrmann, 2011; Bolle et al., 2014; Charness et al., 2014; Abbink et al., 2021), a formal analysis is lacking.

This paper provides a game-theoretic analysis of social comparison preferences. *First*, we propose a unified model that captures a spectrum of social comparison preferences – from competitive preferences to (imperfect) inequality aversion – while maintaining tractability to generate sharp predictions. *Second*, we identify a novel property called decomposability and show that many classical games central to understanding workplace productivity and behavior in social dilemmas are decomposable games. Our key insight is that under social comparison preferences, every decomposable game reduces to a collection of non-strategic individual decision problems, yielding a unique prediction in generic settings. *Third*, we apply our approach to demonstrate a social comparison trap: it drives individuals to exert the minimum effort in the minimum effort game, contribute the least in the public goods game, and over-extract to the maximum in the tragedy of the commons.

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We consider a general n-person normal-form game of complete information. Given material payoffs in the original game, a player's social comparison preferences are represented by a reference-dependent utility function, which is a function of the player's relative payoffs to a social reference point. In our main analysis, this social reference point is the average payoff of all players. Our model can represent *competitive preferences* or *imperfect inequality aversion*. A competitive player seeks to gain relative advantages as much as possible, so that the player's utility increases with their payoff advantage relative to the social reference point universally. An imperfectly inequality-averse player dislikes being worse off than others or substantially ahead; their bliss point is to be slightly ahead of others.[1] Thus, only when their advantage relative to others is less than a threshold that represents their ideal superiority level does their utility increase with their advantage.[2]

A social comparison equilibrium is a Nash equilibrium in games under social comparison preferences. We characterize these equilibria in decomposable games. A game is *decomposable* if each player's material payoff function can be expressed as the sum of (i) a common component shared by all players and (ii) an idiosyncratic component that depends only on the player's own strategy. Decomposable games are a subclass of potential games (Monderer and Shapley, 1996), encompassing all 2×2 normal-form symmetric games as well as diverse multi-player, multi-strategy games such as the minimum effort game, the public goods game, and the tragedy of the commons.

We show that a social comparison equilibrium in a decomposable game is the collection of the maximum strategies of all players' idiosyncratic components. This implies that, *first*, a generically unique social comparison equilibrium exists in every decomposable game whenever each player's idiosyncratic component has a maximum. *Second*, social comparison equilibria are easy to compute, by maximizing each player's idiosyncratic component that depends solely on their own strategy. *Third*, a social comparison equilibrium need not be a Nash equilibrium of the original game. In other words, social comparison may lead to very different outcomes compared to the standard analysis of games with material payoff maximizers.

Social comparison equilibria often result in negative outcomes. Under social comparison preferences, when players evaluate their payoffs relative to the average payoffs, they systematically ignore the common component of payoffs that affects all players equally. This implies that when forming best responses, they will not only ignore the positive externalities on others but also be willing to forgo benefits for themselves to increase their economic status. Consequently, social comparison can lead to outcomes that are more detrimental for everyone than any Nash equilibrium in the original game with material payoff maximizers. We term this phenomenon the *social comparison trap*. This social comparison trap is not limited to competitive preferences. Even when all players only seek arbitrarily mild superiority over others, due to the process of competitive escalation, the generically unique social comparison equilibrium may still result in the same negative outcome as that under competitive preferences.

This dark side of social comparison is not merely theoretical: a number of studies in economics and psychology have documented that individuals may engage in destructive behavior to improve their relative standing without monetary gains, sometimes even when they are already in an advantageous position (Zizzo and Oswald, 2001; Herrmann et al., 2008; Abbink and Herrmann, 2011; Bolle et al., 2014; Charness et al., 2014), especially in insecure environments (Gilbert et al., 2009; Abbink and De Haan, 2014; Abbink et al., 2021).[3]

1.1. Related literature

This study develops a simple model of social comparison preferences and identifies the class of decomposable games for which sharp predictions can be obtained, thus contributing to several strands of the literature in economics and the social sciences.

First, this study contributes to the broad literature on social comparison and status seeking (e.g., Frank, 1987; Hopkins and Kornienko, 2004; Luttmer, 2005; Card et al., 2012). While empirical studies have documented the importance of social comparison in various contexts, frameworks for analyzing its strategic implications have been limited. Recent advances include Arrow and Dasgupta (2009) and Cres et al. (1997), and studies by Immorlica et al. (2017), Ghiglino and Goyal (2010), and Cheng and Xing (2022) on social comparison and conspicuous consumption within networks.[4] We contribute by providing a unified characterization of social comparison equilibrium in a class of games via decomposability.

Second, we contribute to the literature on other-regarding preferences. Our model of social comparison preferences aligns with theoretical work such as Rabin (1993), Fehr and Schmidt (1999), and Battigalli and Dufwenberg (2007). Our tractable model captures social comparison preferences and nests competitive preferences and imperfect inequality aversion as special cases. While some predictions may seem extreme, they offer benchmarks for guiding intuition.

In particular, the case of imperfect inequality aversion is related to inequality aversion (Fehr and Schmidt, 1999; Bolton and Ockenfels, 1999; Engelmann and Strobel, 2004; Kets et al., 2011), which has been extensively studied both theoretically and experimentally over the past two decades. We find that even when players seek arbitrarily mild superiority, it can result in dramatically different and much more negative outcomes across various settings compared to the preference for exactly equalizing payoffs.

The social comparison trap may be relevant in understanding empirical puzzles such as persistent coordination failures observed in laboratory experiments (Van Huyck et al., 1990; Weber, 2006), imperfect conditional cooperation in repeated public goods game experiments (Fischbacher and Gachter, 2010), moody conditional cooperators in spatial Prisoner's dilemma game experiments (Grujic et al., 2014), and the use of extortionate strategies in repeated social dilemma game experiments (Milinski et al., 2016).[5] Moreover, it suggests that policies aimed at improving social outcomes must account for how relative payoff concerns can undermine efficiency.

Third, our study extends the potential games literature (Monderer and Shapley, 1996). Traditional studies of potential games have focused on convergent and equilibrium properties of the original game without considering further implications of social preferences. We show how social comparison preferences within a subclass of potential games systematically alter strategic behavior and produce inefficient outcomes in these well-established theoretical structures.

[1] They may also be called limited superiority seeking players.

[2] When this ideal superiority level converges to zero, imperfect inequality aversion converges to (perfect) inequality aversion where players prefer exactly equalizing payoffs with others.

[3] The finding that competitive preferences and destructive behavior are particularly salient when individuals feel insecure or depressed suggests that such preferences may be context-dependent. This empirical question regarding the domain of social comparison preferences warrants future research.

[4] In Section 6.2 we extend the model to address social comparison within networks and show that all main results continue to hold.

[5] In a repeated public goods game, imperfect conditional cooperators contribute slightly lower than the group average. As a result, they earn slightly higher than the group average. In a spatial Prisoner's dilemma game, moody conditional cooperators make decisions based on their own previous actions and the actions of their neighbors. Specifically, they behave as imperfect conditional cooperators (i.e., the probability of cooperating is slightly lower than the fraction of cooperating neighbors) when they cooperated in the previous round (Grujic et al., 2014). In a social dilemma game, a player using an extortionate strategy can guarantee that its own payoff is never smaller than the opponent's payoff. In several experiments, humans adopt extortionate strategies in repeated Prisoner's dilemma games and collective-risk games, even when extortionate strategies do not appear in any Nash equilibrium (Milinski et al., 2016).

Finally, our study extends the literature on reference-dependent preferences. While the literature usually focuses on individual decision-making under uncertainty (see e.g., Kahneman and Tversky (1979), Tversky and Kahneman (1991) and O'Donoghue and Sprenger (2018) for a survey), we focus on strategic settings and consider a summary statistic of the relevant group as a reference point. Thus, our model is more related to the recent work by Nax and Newton (2019) and Sawa and Wu (2018a,b), which also examine reference-dependence in games. Our novelty lies in uncovering the connections between decomposable games and social comparison preferences and elaborating the social comparison trap.

The paper is organized as follows. Section 2 presents the model for social comparison preferences. Section 3 introduces decomposability. Section 4 presents our main results. Section 5 applies our results to several classic games, demonstrating the social comparison trap. Section 6 discusses several extensions, including alternative reference points, social comparison within networks, and a model that trades off social comparison against payoff maximization. Section 6 also briefly discusses social comparison equilibrium in non-decomposable games.

2. Social comparison preferences

Consider an n-person non-cooperative game in normal form with complete information. The set of players is denoted by $N = \{1, 2, \ldots, n\}$, where $n \geq 2$. For each player $i \in N$, the set of pure strategies is denoted by S_i, with $|S_i| \geq 2$. A strategy profile is denoted by $s = (s_1, \ldots, s_n) \in S = \prod_{j \in N} S_j$. Let $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \in S_{-i} = \prod_{j \in N \setminus \{i\}} S_j$ be a strategy profile of all players except i.

The material payoffs of player i are specified by a function $\pi_i : S \mapsto \mathbb{R}$. Given others' strategies s_{-i}, player i's material payoff from playing s_i is $\pi_i(s_i, s_{-i})$. We refer to the game with material payoffs as the *material game*, denoted by $\Gamma = (N, \{S_i\}_{i \in N}, \{\pi_i\}_{i \in N})$.

Building on the extensive evidence from psychology that individuals evaluate their outcomes relative to others, we propose a utility function to represent players' social comparison preferences. It has two features. First, players derive utility from comparing their own material payoffs relative to a social reference point. In our main analysis, this social reference point is the average payoff of the reference group.[6] Second, the model nests two prominent social preferences – inequality aversion and competitive preferences – as limit cases.

Formally, given a material game $\Gamma = (N, \{S_i\}_{i \in N}, \{\pi_i\}_{i \in N})$, we consider the *extended game* $\widetilde{\Gamma} = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ with utility functions:

$$u_i(s) = v\Big(\pi_i(s) - r_i(s)\Big) \tag{1}$$

where $v(\cdot)$ is called the *value function*, and the *social reference point*

$$r_i(s) = \sum_{j \in N} \frac{\pi_j(s)}{n} \tag{2}$$

averages players' material payoffs.

We impose the following structure on the value function $v(\cdot)$, which encompasses inequality aversion and competitive preferences as limit cases:

Assumption 1. There exists a critical threshold $\tau \geq 0$ such that $v(\cdot)$ is strictly increasing on $(-\infty, \tau]$ while decreasing on $[\tau, +\infty]$.

When τ goes to zero, players' preferences converge to one for minimizing payoff differences between their own and the group average as much as possible. We refer to the limit case $\tau = 0$ as (perfect) inequality aversion.[7] When τ approaches infinity, the utility function captures competitive preferences: players prefer to be as far ahead of others as possible.

When $0 < \tau < +\infty$, we have intermediate cases: players dislike being worse off than others or being substantially better off; instead, their ideal position is ahead of others by the magnitude of τ. We find these intermediate cases intuitive and plausible, particularly those with a small $\tau > 0$, where players seek mild superiority over others. However, we are not aware of any existing work formalizing this scenario. To better discuss these scenarios and highlight their differences from (perfect) inequality aversion ($\tau = 0$) and competitive preferences (sufficiently large τ), we refer to the cases of small $\tau > 0$ as *imperfect inequality aversion*.[8]

We aim to characterize the Nash equilibria of the extended game $\widetilde{\Gamma} = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$, which are referred to as *social comparison equilibria (SCE)*.

Remarks. *First*, our specification of the social comparison utility function (1) follows the classical work on reference-dependence in decisions under uncertainty (e.g., Kahneman and Tversky, 1979; Tversky and Kahneman, 1991) and the recent work by Nax and Newton (2019) and Sawa and Wu (2018a) in games. However, since our analysis does not involve expected utility calculations, our results do not depend on the curvature of $v(\cdot)$.

Second, our utility function implies that players may engage in self-destructive behavior to increase their relative payoffs, particularly when they are in an inferior position. Classical experimental evidence of rejecting unfair offers in ultimatum games and punishing free riders in public goods games is often attributed to the motive of minimizing payoff differences or reciprocity (Camerer, 2003). However, these actions are also consistent with competitive preferences. Moreover, our model can explain destructive behaviors that cannot be explained by inequality aversion (Fehr and Schmidt, 1999; Bolton and Ockenfels, 1999) or fairness preferences (Rabin, 1993; Dufwenberg and Kirchsteiger, 2004), such as attacks and arms races in the strike games (Abbink and De Haan, 2014; Abbink et al., 2021), reducing disadvantageous others' payoffs (Charness and Grosskopf, 2001; Abbink et al., 2011), sabotage to increase ranking without monetary gains (Charness et al., 2014), the joy of being nasty (Abbink and Sadrieh, 2009), and antisocial punishment in the public goods game (Herrmann et al., 2008).[9]

Third, when τ goes to zero, our utility function converges to one representing (perfect) inequality aversion. Our model is thus related to Fehr and Schmidt (1999) (FS) and Bolton and Ockenfels (1999). In FS, similar to a typical social preferences model, players' utility function includes a self-payoffs term π_i in addition to the term for reducing payoff differences. Our primary departure, however, lies not in the functional form or whether players care about their own material payoffs. In our model, when $\pi_i - r_i < \tau$, holding other players' payoffs constant, a player's utility increases with π_i as in FS. Rather, the key difference is the willingness to pay for reducing or increasing others' payoffs. Given their assumptions on parameters, the willingness to pay for reducing others' payoffs when $\pi_i - r_i < \tau$ and that for increasing others' payoffs when $\pi_i - r_i > \tau$ are *both greater* in our model.[10] Therefore, it is not immediately obvious whether our model

[6] Section 6 discusses alternative social reference points and their implications.

[7] We discuss the connections with Fehr and Schmidt (1999) below.

[8] An alternative description for these cases would be limited superiority seeking.

[9] Studies show that the extent of destructive behavior depends on the framing (Abbink et al., 2011) and that competitive motives may be more salient when individuals feel insecure (Gilbert et al., 2009). Therefore, competitive motives may be context-dependent.

[10] In FS, a player would never want to increase another player's payoffs at the cost of reducing their own payoffs by the same magnitude, whereas this can happen in our model. We further discuss the implications of this in Section 6.4. We also differ in the reference point for comparison. In FS, players compare their payoffs with the payoffs of each other player independently.

of imperfect inequality aversion would lead to less or more cooperative outcomes than FS.

3. Decomposable games

To obtain sharp predictions, we focus on the class of decomposable games:[11]

Definition 1 (*Decomposable games*). The material game $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ is *decomposable* if there exist a function $\phi : S \mapsto \mathbb{R}$ common to all players and an idiosyncratic function $\delta_i : S_i \mapsto \mathbb{R}$ for each $i \in N$ such that $\pi_i(s) = \phi(s) + \delta_i(s_i)$ for all $i \in N$ and $s \in S$.

In other words, if a game is decomposable, then the material payoffs $\pi_i(s)$ can be expressed as the sum of a *common component* $\phi(s)$ and an *idiosyncratic component* $\delta_i(s_i)$, where the idiosyncratic component depends only on the player's own strategy. If this holds, we say the game is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$.

Decomposable games have three immediate implications. First, any strategy change of a player i will affect another player j's material payoffs only through the common component $\phi(s)$, and these spillover effects are identical across all $j \in N \setminus \{i\}$. Second, if we subtract the common spillovers from a player's material payoffs, then the residuals depend only on the player's own strategy: $\pi_i(s) - \phi(s) = \delta_i(s_i)$. Third, any pairwise payoff comparisons depend only on the idiosyncratic components, which in turn depend only on the strategies of the players involved:

$$\pi_i(s) - \pi_j(s) = \delta_i(s_i) - \delta_j(s_j). \tag{3}$$

We show in Section 4 that these properties yield sharp predictions in games with social comparison preferences.

3.1. Examples

Decomposable games include a variety of classical games that have received wide attention in the literature. We provide four examples in the following.[12,13]

Example 1 (*2 × 2 coordination games*). Consider 2 × 2 coordination games as follows:

	A	B
A	a, a	c, d
B	d, c	b, b

where $a > d$ and $b > c$ so that both (A, A) and (B, B) are strict Nash equilibria. In addition, we assume $b > a$ so that the equilibrium (B, B) is payoff-dominant. We also assume $a > c$, implying that an A-player prefers playing against another A-player over playing against a B-player.[14]

This game is decomposable with

$\phi(A, A) = a,\ \phi(A, B) = \phi(B, A) = c$ and $\phi(B, B) = b + c - d.$

and

$\delta_1(A) = \delta_2(A) = 0$ and $\delta_1(B) = \delta_2(B) = d - c.$

Furthermore, the following example examines an alternative class of coordination games.

Example 2 (*Minimum-effort game*). Consider the minimum effort game, also called the weakest-link game in the literature, introduced by Van Huyck et al. (1990). Players choose their effort levels from the common set $E = \{e^1, \dots, e^\rho\}$, where $\rho \geq 2$ is an integer and $0 < e^1 < \cdots < e^\rho < +\infty$. For any $e = (e_1, \dots, e_n) \in E^n$ and $i \in N$, the material payoff of player i is given by

$$\pi_i^{MEG}(e_1, \dots, e_n) = \min_{j\in N} e_j - \delta e_i$$

where $0 < \delta < 1$.

The minimum effort game is decomposable, where the common component is given by $\min_{j\in N} e_j$, and the idiosyncratic component is given by $-\delta e_i$ for all players $i \in N$.

Note that for the material payoff functions in Example 2, if the min operation is replaced by any other order statistic of $e_1, \dots, e_n$, we can obtain another class of decomposable material games. In particular, if we use the max operation, we have

$$\pi_i(e_1, \dots, e_n) = \max_{1\leq j\leq n} e_j - \delta e_i$$

where $0 < \delta < 1$. It is straightforward that in order to maximize material payoffs, (i) if some opponent chooses e^ρ, it is optimal for player i to choose e^1, and (ii) if none of the opponents chooses e^ρ, it is optimal for player i to choose e^ρ, provided that δ is small.[15] Therefore, we obtain a class of public goods games.

The following example presents an alternative class of public goods games that may be more familiar to readers.

Example 3 (*Public goods game*). Consider public goods game as in Bramoullé and Kranton (2007). Let $e_i \in [0, +\infty)$ denote player i's level of effort. For any $e = (e_1, \dots, e_n) \in [0, +\infty)^n$, the material payoff of player i is given by

$$\pi_i^{PGG}(e_1, \dots, e_n) = b\Big(\sum_{j\in N} e_j\Big) - ce_i$$

where (i) $b(\cdot)$ is a twice differentiable function satisfying that $b(0) = 0$, $b' > 0$, and $b'' < 0$ and (ii) $c > 0$ and $b'(0) > c > \lim_{e\to+\infty} b'(e)$.

The public goods game is decomposable, where the common component is given by $b\big(\sum_{j\in N} e_j\big)$, and the idiosyncratic component is given by $-ce_i$ for all players $i \in N$.

The common component in either of Examples 2 and 3 is an increasing function of all players' strategic choices. This monotonicity captures positive externalities of players' strategic choices. The following example serves as a contrast and takes into account the possibility of negative externalities of players' strategic choices.

Example 4 (*Tragedy of the commons*). Consider the tragedy of the commons game, in the spirit of Hardin (1968). Each player i chooses their own consumption level of some resource $k_i \in [0, K]$. For any $k = (k_1, \dots, k_n) \in [0, K]^n$, the material payoff of player i is given by

$$\pi_i^{TCG}(k_1, \dots, k_n) = \ln\Big(nK + 1 - \sum_{j\in N} k_j\Big) + \ln\big(1 + k_i\big).$$

[11] The functions $\phi(\cdot)$ and $\delta_i(\cdot)$ depend on the material game Γ. For simplicity, we omit this dependence in the notations.

[12] These games are extensively studied in the literature on both experimental and theoretical economics. For 2 × 2 coordination games, see, e.g., Kandori et al. (1993), Young (1993), Sawa and Wu (2018a,b), and Nax and Newton (2019). For minimum effort games, see, e.g., Van Huyck et al. (1990), Alós-Ferrer and Weidenholzer (2014), and Riedl et al. (2016). For public goods games, see, e.g., Bramoullé and Kranton (2007) and Galeotti and Goyal (2010). For the tragedy of the commons games, see, e.g., Hardin (1968).

[13] All of these examples are symmetric games. However, it is easy to construct a decomposable game that is not symmetric. For instance, in the public goods games in Example 3, different players can have different marginal costs of exerting efforts.

[14] These two assumptions are introduced to facilitate discussion later. In fact, the analysis in this example can be extended to all 2 × 2 symmetric games.

[15] When none of the opponents takes e^ρ, if player i chooses e^ρ, their material payoff will be $e^\rho - \delta e^\rho$, and if player i chooses another effort level, rather than e^ρ, their material payoff will be at most $e^{\rho-1} - \delta e^1$. It is optimal for player i to choose e^ρ, provided that $\delta < \frac{e^\rho - e^{\rho-1}}{e^\rho - e^1}$.

The tragedy of the commons game is decomposable, where the common component is given by $\ln(nK + 1 - \sum_{j\in N} k_j)$, and the idiosyncratic component is given by $\ln(1 + k_i)$ for all players $i \in N$.

3.2. General properties

We present several general properties of decomposable games in this subsection. First, we note that decomposable games are a subclass of potential games (Monderer and Shapley, 1996), which implies that all the properties of potential games discussed in the literature apply to decomposable games.[16] Consider a game Γ decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$. Define a function $P : S \mapsto R$ such that

$$P(s) = \phi(s) + \sum_{i\in N} \delta_i(s_i). \tag{4}$$

Proposition 1. *If the material game $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ is decomposable, then it admits $P(s)$ as an exact potential function such that for each $s' = (s_i', s_{-i})$ and $s = (s_i, s_{-i})$, we have $P(s') - P(s) = \pi_i(s') - \pi_i(s)$. Furthermore, when $n = 2$, the game Γ is decomposable if and only if it admits an exact potential.*

Proof. The first statement that a decomposable game admits $P(s)$ as an exact potential follows directly from $\pi_i(s) = \phi(s)+\delta_i(s_i)$. In the following we prove that an exact potential game with a potential function $P(s)$ is a decomposable game when $n = 2$.

Suppose that Γ is an exact potential game with potential function $P(s)$, and that s_1^* and s_2^* are two arbitrarily chosen strategies for player 1 and player 2, respectively. Then, based on the definition of potential function, we have $\pi_1(s_1, s_2) = \pi_1(s_1^*, s_2) + P(s_1, s_2) - P(s_1^*, s_2)$ and $\pi_2(s_1, s_2) = \pi_2(s_1, s_2^*) + P(s_1, s_2) - P(s_1, s_2^*)$. Let us define $\phi(s_1, s_2) = P(s_1, s_2) - P(s_1^*, s_2) - P(s_1, s_2^*) + \pi_1(s_1^*, s_2) + \pi_2(s_1, s_2^*)$, $\delta_1(s_1) = P(s_1, s_2^*) - \pi_2(s_1, s_2^*)$, and $\delta_2(s_2) = P(s_1^*, s_2) - \pi_1(s_1^*, s_2)$. It is easy to check $\pi_1(s_1, s_2) = \phi(s_1, s_2) + \delta_1(s_1)$ and $\pi_2(s_1, s_2) = \phi(s_1, s_2) + \delta_2(s_2)$. That is, the game is decomposable. □

Second, it follows directly from the definition that if a game is decomposable, then the payoff differences between any two players depend only on the strategies of these two players and not on others. Proposition 2 below shows that the converse also holds for any game with $n \geq 3$: if the material payoff differences between any two players depend only on the strategies of the players involved, then the game is decomposable. More precisely, for any two distinct players i and j, $i < j$, let $s_{-ij} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_{j-1}, s_{j+1}, \ldots, s_n) \in S_{-ij} = \prod_{k\in N\setminus\{i,j\}} S_k$ denote a strategy profile of all players except i and j.

Proposition 2. *Suppose $n \geq 3$. Then the material game $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ is decomposable if and only if for all distinct $i, j \in N$, the payoff differences $\pi_i(\cdot) - \pi_j(\cdot)$ are invariant to s_{-ij}.*

Proof. We focus only on the sufficiency part. The proof of the necessity part is straightforward and we omit it.

Consider any three distinct players i, j, and k. Note that the difference $\pi_i(\cdot) - \pi_k(\cdot)$ depends only on s_i and s_k, the strategic choices of players i and k, and that the difference $\pi_j(\cdot) - \pi_k(\cdot)$ depends only on s_j and s_k, the strategic choices of players j and k. For convenience, we denote $\pi_i(s) - \pi_k(s) = f(s_i, s_k)$ and $\pi_j(s) - \pi_k(s) = g(s_j, s_k)$.

We first show that $\pi_i(s)$ and $\pi_j(s)$ have the forms $\pi_i(s) = \phi(s)+\delta_i(s_i)$ and $\pi_j(s) = \phi(s) + \delta_j(s_j)$. By rearranging terms, $\pi_i(s) - \pi_j(s) = (\pi_i(s) - \pi_k(s)) - (\pi_j(s) - \pi_k(s)) = f(s_i, s_k) - g(s_j, s_k)$. Note that $\pi_i(\cdot) - \pi_j(\cdot)$ depends only on s_i and s_j and is invariant in s_k. Let us take $s_k = s_k^*$, where s_k^* is an arbitrarily chosen strategy of player k, and denote $\delta_i(s_i) = f(s_i, s_k^*)$, $\delta_j(s_j) = g(s_j, s_k^*)$. Then, we have $\pi_i(s)-\pi_j(s) = \delta_i(s_i)-\delta_j(s_j)$; equivalently, $\pi_i(s) - \delta_i(s_i) = \pi_j(s) - \delta_j(s_j)$. Denote this common term by $\phi(s)$. As a result, $\pi_i(s)$ and $\pi_j(s)$ can be written as $\pi_i(s) = \phi(s) + \delta_i(s_i)$ and $\pi_j(s) = \phi(s) + \delta_j(s_j)$, respectively.

We next show that $\pi_k(s)$ also has the form $\pi_k(s) = \phi(s) + \delta_k(s_k)$. Following the same reasoning as above, there exist $\delta_i'(s_i)$, $\delta_k'(s_k)$, and $\phi'(s)$ such that $\pi_i(s) = \phi'(s) + \delta_i'(s_i)$ and $\pi_k(s) = \phi'(s) + \delta_k'(s_k)$. Hence, it must be true that $\phi(s) + \delta_i(s_i) = \phi'(s) + \delta_i'(s_i)$. By simple calculation, $\pi_j(s)-\pi_k(s) = \phi(s)+\delta_j(s_j)-\phi'(s)-\delta_k'(s_k) = \delta_j(s_j)-\delta_k'(s_k)+\delta_i'(s_i)-\delta_i(s_i)$, where the second equality follows from the fact that $\phi(s) + \delta_i(s_i) = \phi'(s) + \delta_i'(s_i)$. Note also that $\pi_j(s) - \pi_k(s)$ is independent of s_i. This implies that $\delta_i'(s_i) - \delta_i(s_i)$ must be a constant, denoted by C. Thus, $\phi(s) - \phi'(s) = C$, and we get $\pi_k(s) = \phi'(s) + \delta_k'(s_k) = \phi(s) + \delta_k'(s_k) - C$. Denoting $\delta_k(s_k) = \delta_k'(s_k) - C$, then we have $\pi_k = \phi(s) + \delta_k(s_k)$. □

Hence, when $n \geq 3$, to verify decomposability, it is sufficient to check whether all pairwise payoff differences are independent of all other players' strategies. However, when $n = 2$, this pairwise comparison criterion does not guarantee decomposability, since the condition imposes no restriction on 2-person games. [17]

Third, if a game is decomposable, then all its decompositions are unique up to a constant.

Proposition 3. *If a game Γ is decomposable with both $\{\phi, \{\delta_i\}_{i\in N}\}$ and $\{\phi', \{\delta_i'\}_{i\in N}\}$, then there is a constant $c \in \mathbb{R}$ such that for all $s \in S$ and $i \in N$,*

$$\phi'(s) = \phi(s) + c \quad \textit{and} \quad \delta_i'(s) = \delta_i(s) - c.$$

Proof. It is sufficient to focus on the uniqueness of the common component. The uniqueness of the idiosyncratic components follows directly from the uniqueness of the common component.

Suppose that Γ is decomposable both with the common component ϕ and the idiosyncratic components $\{\delta_i\}_{i\in N}$ and with the common component ϕ' and the idiosyncratic components $\{\delta_i'\}_{i\in N}$. By Definition 1, for all players $i \in N$, both the following two equations hold:

$$\pi_i(s) = \phi(s) + \delta_i(s_i) \text{ for all } s \in S$$

and

$$\pi_i(s) = \phi'(s) + \delta_i'(s_i) \text{ for all } s \in S.$$

By simple calculation, it then follows that

$$\phi'(s) - \phi(s) = \delta_i(s_i) - \delta_i'(s_i) \text{ for all } s \in S. \tag{5}$$

By Eq. (5), for any two distinct players i and j, $\phi' - \phi$ is a function of only s_i, the strategic choice of i, and at the same time, it is a function of only s_j, the strategic choice of j. Therefore, it has to be true that there exists some constant $c \in \mathbb{R}$ such that $\phi'(\cdot) - \phi(\cdot) = c$. □

Therefore, while a decomposable game may have multiple representations, these representations differ only by a constant shift between the common and idiosyncratic components. This uniqueness property

[16] For instance, every exact potential game with a finite strategy set possesses a pure-strategy Nash equilibrium, and the asynchronous best-response dynamics converge to a pure-strategy Nash equilibrium in finite steps (Monderer and Shapley, 1996).

[17] For instance, consider the following 2×2 game:

	A	B
A	1, 2	0, 0.5
B	1, 1.5	2, 4

Suppose this game is decomposable. Without loss of generality, suppose that $\phi(A, A) = 1$. By $\pi_1(A, A) = 1$ and $\pi_2(A, A) = 2$, $\delta_1(A) = 0$ and $\delta_2(A) = 1$. Then, by $\pi_1(A, B) = 0$ and $\delta_1(A) = 0$, $\phi(A, B) = 0$, which implies that $\delta_2(B) = 0.5$, in combination with $\pi_2(A, B) = 0.5$. Further, by $\pi_2(B, A) = 1.5$ and $\delta_2(A) = 1$, $\phi(B, A) = 0.5$, which implies that $\delta_1(B) = 0.5 = \delta_2(B)$, in combination with $\pi_1(B, A) = 1$. As a result, we should have $\pi_1(B, B) = \pi_2(B, B)$, which yields a contradiction. Furthermore, it can be shown that a 2×2 game is decomposable if and only if $[\pi_1(A, A) + \pi_1(B, B)] - [\pi_1(A, B) + \pi_1(B, A)] = [\pi_2(A, A) + \pi_2(B, B)] - [\pi_2(A, B) + \pi_2(B, A)]$, provided that two players have the common set of pure strategies, $\{A, B\}$.

ensures that our results are invariant to the specific decomposition chosen.

Finally, any linear combination of decomposable games is decomposable. Formally, consider two n-person material games $\Gamma' = (N, \{S_i\}_{i\in N}, \{\pi'_i\}_{i\in N})$ and $\Gamma'' = (N, \{S_i\}_{i\in N}, \{\pi''_i\}_{i\in N})$. We say that the material game $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ is a *linear combination* of Γ' and Γ'' if there exist some non-zero scalars $\alpha, \beta \in \mathbb{R}$ such that $\pi_i(s) = \alpha\pi'_i(s)+\beta\pi''_i(s)$ for each $s \in S$ and $i \in N$. We denote by $\alpha\Gamma'+\beta\Gamma''$ the linear combination of Γ' and Γ''.

Proposition 4. *If the material games Γ' and Γ'' are decomposable, then any linear combination of them is decomposable.*

The proof is simple. Suppose that the game $\Gamma' = (N, \{S_i\}_{i\in N}, \{\pi'_i\}_{i\in N})$ is decomposable with $\{\phi', \{\delta'_i\}_{i\in N}\}$ and the game $\Gamma'' = (N, \{S_i\}_{i\in N}, \{\pi''_i\}_{i\in N})$ is decomposable with $\{\phi'', \{\delta''_i\}_{i\in N}\}$. Then, for a linear combination of them $\Gamma = \alpha\Gamma' + \beta\Gamma''$, we have material payoffs

$$\pi_i(s) = \alpha\pi'_i(s) + \beta\pi''_i(s) = \Big(\alpha\phi'(s) + \beta\phi''(s)\Big) + \Big(\alpha\delta'_i(s_i) + \beta\delta''_i(s_i)\Big).$$

Thus, the game Γ is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$, where $\phi(\cdot) = \alpha\phi'(\cdot) + \beta\phi''(\cdot)$ and $\delta_i(\cdot) = \alpha\delta'_i(\cdot) + \beta\delta''_i(\cdot)$ for each $i \in N$. Consequently, once we obtain two decomposable games, we can construct a family of them by taking linear combinations.

4. Main results

Our analysis starts with the following observation. If the underlying material game Γ is decomposable with the common component ϕ and the idiosyncratic components $\{\delta_i\}_{i\in N}$, then the difference $\Delta_i(s_i, s_{-i}) \equiv \pi_i(s_i, s_{-i}) - r_i(s_i, s_{-i})$ can be rewritten as

$$\begin{aligned}\Delta_i(s_i, s_{-i}) &= \pi_i(s_i, s_{-i}) - \frac{1}{n}\sum_{j\in N}\pi_j(s_i, s_{-i}) &(6)\\ &= \Big(\phi(s) + \delta_i(s_i)\Big) - \frac{1}{n}\sum_{j\in N}\Big(\phi(s) + \delta_j(s_j)\Big)\\ &= \Big(\phi(s) + \delta_i(s_i)\Big) - \Big(\phi(s) + \frac{1}{n}\sum_{j\in N}\delta_j(s_j)\Big)\\ &= \delta_i(s_i) - \frac{1}{n}\sum_{j\in N}\delta_j(s_j)\\ &= \Big(1-\frac{1}{n}\Big)\delta_i(s_i) - \frac{1}{n}\sum_{j\neq i}\delta_j(s_j), &(7)\end{aligned}$$

where the first equality follows from $r_i(\cdot) = \frac{\sum_{j\in N}\pi_j(\cdot)}{n}$, and the second one follows from the decomposability of the material game Γ. In other words, given that player i has the average material payoff as the reference point, the difference between their material payoff and the reference point is determined by the idiosyncratic components only, because the common component $\phi(\cdot)$ belongs to both their self payoff term π_i and the reference point r_i, and can be offset.

Armed with the above observation, we examine the consequences of competitive preferences (sufficiently large τ), imperfect inequality aversion (small τ), and heterogeneous players (only some players concerned with social comparison) in what follows.

4.1. Competitive preferences

Recall that the utility function of player i is given by $u_i(s_i, s_{-i}) = v(\pi_i(s_i, s_{-i}) - r_i(s_i, s_{-i}))$, where the value function $v(\cdot)$ is strictly increasing on $(-\infty, \tau]$. Therefore, given a sufficiently large τ, the problem of choosing $s_i \in S_i$ to maximize $u_i(s_i, s_{-i})$ is equivalent to the problem of maximizing $\Delta_i(s_i, s_{-i}) = \pi_i(s_i, s_{-i}) - r_i(s_i, s_{-i})$ given any $s_{-i} \in S_{-i}$. Note further that $1 - 1/n \geq 1 - 1/2 > 0$, and that the term $\frac{\sum_{j\in N\setminus\{i\}}\delta_j(s_j)}{n}$ is independent of s_i. Thus, the problem of maximizing $u_i(s_i, s_{-i})$ is reduced to the problem of maximizing $\delta_i(s_i)$. Consequently, for each $s_{-i} \in S_{-i}$,

$$\arg\max_{s'_i\in S_i} u_i(s'_i, s_{-i}) = \arg\max_{s'_i\in S_i} \Delta_i(s'_i, s_{-i}) = \arg\max_{s'_i\in S_i} \delta_i(s'_i) \tag{8}$$

provided that $\Delta_i(s_i, s_{-i}) \leq \tau$ for all $s_i \in S_i$. Hence, in the extended game $\widetilde{\Gamma}$, the best-response strategies of player i are completely determined by the idiosyncratic component $\delta_i(\cdot)$ of their material payoff, independent of the strategic choices s_{-i} of other players.[18] This fact plays an important role in characterizing social comparison equilibria.

Based on the above analysis, the following theorem provides a sufficient and necessary condition for the existence of social comparison equilibria.

Theorem 1. *Suppose Γ is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$, and $\tau \geq 2\max_{i\in N}\max_{s_i\in S_i}|\delta_i(s_i)|$. Then an SCE exists if and only if $\arg\max_{s_i\in S_i}\delta_i(s_i) \neq \emptyset$ for each $i \in N$. Furthermore, the strategy profile $s^* = (s^*_1, \dots, s^*_n)$ is an SCE if and only if $s^*_i \in \arg\max_{s_i\in S_i}\delta_i(s_i)$ for each $i \in N$.*

Proof. Consider the extended game $\widetilde{\Gamma} = (N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N})$ where the underlying material game is $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$. Note that for Γ, the common component is given by $\phi(\cdot)$ and the idiosyncratic components are given by $\{\delta_i(\cdot)\}_{i\in N}$. For every player $i \in N$, $\pi_i(s) = \phi(s) + \delta_i(s_i)$ for all $s = (s_1, \dots, s_i, \dots, s_n) \in S$.

We only focus on the proof of the existence of Nash equilibria of the game $\widetilde{\Gamma}$. The fact that the profile s^* is an SCE if and only if $s^* \in \prod_{i\in N}\arg\max_{s_i\in S_i}\delta_i(s_i)$ can easily be derived from the proof of the existence.

First, we prove the sufficiency part. Consider a strategy profile $s^* = (s^*_1, \dots, s^*_n) \in S$ satisfying that $s^*_i \in \arg\max_{s_i\in S_i}\delta_i(s_i)$ for all $i \in N$. The existence of the profile s^* follows from the fact that $\arg\max_{s_i\in S_i}\delta_i(s_i) \neq \emptyset$ for all $i \in N$. Given that $\tau \geq 2\max_{k\in N}\max_{s_k\in S_k}|\delta_k(s_k)|$, for all $i \in N$ and $s \in S$, we have $\Delta_i(s) \leq |\Delta_i(s)| = \left|(1-\frac{1}{n})\delta_i(s_i) - \frac{\sum_{j\in N\setminus\{i\}}\delta_j(s_j)}{n}\right| \leq (1-\frac{1}{n})|\delta_i(s_i)| + \frac{\sum_{j\in N\setminus\{i\}}|\delta_j(s_j)|}{n} < 2\max_{k\in N}\max_{s_k\in S_k}|\delta_k(s_k)| \leq \tau$. Moreover, since $s^*_i \in \arg\max_{s_i\in S_i}\delta_i(s_i)$ for all $i \in N$, according to Eq. (8), we have

$$s^*_i \in \arg\max_{s'_i\in S_i} u_i(s'_i, s^*_{-i}) \text{ for all } i \in N.$$

It is clear that the profile s^* is a social comparison equilibrium.

Next, we prove the necessity part. Suppose that the profile $s^* = (s^*_1, \dots, s^*_n) \in S$ is a social comparison equilibrium. By the definition of Nash equilibrium, it follows that for all $i \in N$,

$$s^*_i \in \arg\max_{s'_i\in S_i} u_i(s'_i, s^*_{-i}).$$

By applying Equation (8) again, we have that

$$\arg\max_{s'_i\in S_i} u_i(s'_i, s^*_{-i}) = \arg\max_{s'_i\in S_i}\delta_i(s'_i)$$

provided that $\tau \geq 2\max_{k\in N}\max_{s_k\in S_k}|\delta_k(s_k)|$. Following from the above two equations, it holds that $\arg\max_{s_i\in S_i}\delta_i(s_i) \neq \emptyset$ for all $i \in N$. ☐

The logic underlying Theorem 1 is straightforward. By the analysis of the best-response strategies, given the decomposability of the material game, when all players have the average material payoffs as the reference points, the best-response strategies of every player are determined by their idiosyncratic components, provided that the bliss point τ is high. Then, both the existence and the characterization of social comparison equilibria are also determined by the properties of the idiosyncratic components. The driving force is that when a player compares herself to others, only their idiosyncratic component matters, because the common component appears in all players' material payoff functions and can be offset after the comparison. In short, when comparing, players do not care about the common component of the material payoff functions.

Theorem 1 immediately implies that if each player's idiosyncratic component has a unique maximizer, then the game has a unique social comparison equilibrium, which is the profile of the maximum strategies of all players' idiosyncratic components.

[18] This result is established based on the decomposability of the underlying material game Γ, rather than the specific form of the value function v.

Corollary 1. *Suppose Γ is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$, and $\tau \geq 2\max_{i\in N}\max_{s_i\in S_i}|\delta_i(s_i)|$. The strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is the unique social comparison equilibrium if and only if $s_i^* \in S_i$ is the unique maximizer of $\delta_i(\cdot)$ for each $i \in N$.*

4.2. Imperfect inequality aversion

What if τ is strictly positive but small — that is, when players have imperfect inequality aversion? Consider a material game Γ that is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$. Let $\bar{\delta}_i \equiv \max_{s_i\in S_i}\delta_i(s_i)$. The strategy set S_i is said to be *rich* relative to τ if for each $s_i \in S_i$ with $\delta_i(s_i) < \bar{\delta}_i$, there exists another strategy $s_i' \in S_i$ such that $0 < \delta_i(s_i') - \delta_i(s_i) \leq \tau$. That is, a player can always find an alternative strategy that improves their idiosyncratic component by a step less than τ until reaching the idiosyncratic component's maximum value. This richness requirement is satisfied in games with continuous strategy spaces or discrete strategy spaces with sufficiently small increments, provided that the idiosyncratic component is continuous.

Theorem 2. *Suppose that the material game Γ is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$. Suppose $\tau > 0$ and each player's strategy set S_i is rich relative to τ. Also, suppose $|\bar{\delta}_i - \bar{\delta}_j| \leq \tau$ for each $i, j \in N$. Then, the strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is an SCE if and only if $s_i^* \in \arg\max_{s_i\in S_i}\delta_i(s_i)$ for each $i \in N$.*

Proof. According to (7), $\Delta_i(s) = \frac{n-1}{n}\delta_i(s_i) - \frac{1}{n}\sum_{j\neq i}\delta_j(s_j)$. First, we show the sufficiency part (idiosyncratic component maximizers⇒SCE). Consider $s^* = (s_1^*, \ldots, s_n^*)$ such that $\delta_i(s_i^*) = \bar{\delta}_i$ for each $i \in N$. Then, for each $i \in N$ and $s_i \in S_i$, we have

$$\Delta_i(s_i, s_{-i}^*) = \frac{n-1}{n}\delta_i(s_i) - \frac{1}{n}\sum_{j\neq i}\delta_j(s_j^*) \leq \frac{1}{n}\sum_{j\neq i}\left(\bar{\delta}_i - \bar{\delta}_j\right) < \tau$$

where the last inequality follows from $|\bar{\delta}_i - \bar{\delta}_j| \leq \tau$ for each $j \in N$. Hence, $u_i(s_i, s_{-i}^*) = v(\Delta_i(s_i, s_{-i}^*))$ is strictly increasing in $\delta_i(s_i)$ for all $s_i \in S_i$. Therefore, $s_i \in S_i$ maximizes $u_i(s_i, s_{-i}^*)$ if and only if it maximizes $\delta_i(s_i)$. It follows that s^* is an SCE.

Next, we show the necessity part (SCE⇒idiosyncratic component maximizers). Suppose $s^* = (s_1^*, \ldots, s_n^*)$ is an SCE. Let $\bar{N} = \{j \in N | \delta_j(s_j^*) = \bar{\delta}_j\}$. Assume for contradiction that there is some $i \in N$ such that $\delta_i(s_i^*) < \bar{\delta}_i$ so that $N \setminus \bar{N}$ is not empty. Now, let i be the player with the smallest $\delta_i(s_i^*)$ among those in $N \setminus \bar{N}$. Then, $\delta_i(s_i^*) < \bar{\delta}_i$, and

$$\begin{aligned}\Delta_i(s_i^*, s_{-i}^*) &= \frac{1}{n}\sum_{j\in\bar{N}}\left(\delta_i(s_i^*) - \bar{\delta}_j\right) + \frac{1}{n}\sum_{j\notin\bar{N}, j\neq i}\left(\delta_i(s_i^*) - \delta_j(s_j^*)\right)\\ &\leq \frac{1}{n}\sum_{j\in\bar{N}}\left(\delta_i(s_i^*) - \bar{\delta}_j\right). \qquad (9)\end{aligned}$$

Given that S_i is rich relative to τ, there exists a strategy $s_i' \in S_i$ such that $0 < \delta_i(s_i') - \delta_i(s_i^*) \leq \tau$. Suppose i deviates to s_i'. Then

$$\begin{aligned}\Delta_i(s_i', s_{-i}^*) &= \frac{1}{n}\sum_{j\neq i}\left(\delta_i(s_i') - \delta_j(s_j^*)\right)\\ &= \frac{1}{n}\sum_{j\neq i}\left(\delta_i(s_i') - \delta_i(s_i^*) + \delta_i(s_i^*) - \delta_j(s_j^*)\right)\\ &= \frac{n-1}{n}\left(\delta_i(s_i') - \delta_i(s_i^*)\right) + \Delta_i(s_i^*, s_{-i}^*) \qquad (10)\\ &\leq \frac{n-1}{n}\left(\delta_i(s_i') - \delta_i(s_i^*)\right) + \frac{1}{n}\sum_{j\in\bar{N}}\left[\delta_i(s_i^*) - \bar{\delta}_j\right] \qquad (11)\\ &= \frac{n-1}{n}\left(\delta_i(s_i') - \delta_i(s_i^*)\right) + \frac{1}{n}\sum_{j\in\bar{N}}\left[\delta_i(s_i') - \left(\delta_i(s_i') - \delta_i(s_i^*)\right) - \bar{\delta}_j\right]\\ &= \frac{n-1-|\bar{N}|}{n}\left(\delta_i(s_i') - \delta_i(s_i^*)\right) + \frac{1}{n}\sum_{j\in\bar{N}}\left[\delta_i(s_i') - \bar{\delta}_j\right]\\ &\leq \frac{n-1-|\bar{N}|}{n}\tau + \frac{|\bar{N}|}{n}\tau \qquad (12)\\ &< \tau,\end{aligned}$$

where inequality (11) follows from (9), and inequality (12) follows from the fact that $\delta_i(s_i') - \bar{\delta}_j \leq \bar{\delta}_i - \bar{\delta}_j \leq |\bar{\delta}_i - \bar{\delta}_j| \leq \tau$. Moreover, from Eq. (10) above, we observe $\Delta_i(s_i', s_{-i}^*) > \Delta_i(s_i^*, s_{-i}^*)$. Thus, given $v(\cdot)$ being strictly increasing on $(-\infty, \tau]$, we obtain a profitable deviation:

$$u_i(s_i', s_{-i}^*) = v\left(\Delta_i(s_i', s_{-i}^*)\right) > v\left(\Delta_i(s_i^*, s_{-i}^*)\right) = u_i(s_i^*, s_{-i}^*),$$

which contradicts with our initial equilibrium supposition. Thus, in every SCE, we have $\delta_i(s_i^*) = \bar{\delta}_i = \max_{s_i\in S_i}\delta_i(s_i)$ for each $i \in N$. □

According to Theorem 2, imperfect inequality aversion generates the same equilibrium predictions as competitive preferences. This is due to *competitive escalation*. If any player does not choose the maximum strategy of their idiosyncratic component, then the player with the lowest idiosyncratic component strategy would change their strategy to get slightly ahead of others. When this player increases their idiosyncratic component, another player becomes the new bottom player, who then has the same incentive to advance their position. This creates an escalating dynamics of competition, eventually pushing everyone to choose the maximum strategy of their idiosyncratic component. Consequently, although each player seeks only mild superiority, they end up playing the same equilibrium as if all players were extremely competitive.

The condition $|\bar{\delta}_i - \bar{\delta}_j| \leq \tau$ for all $i, j \in N$ holds for all symmetric games, and the richness condition holds for all games with convex strategy spaces and continuous idiosyncratic components. Thus, we have the following corollary.

Corollary 2. *In all symmetric decomposable games with convex strategy spaces and continuous idiosyncratic components, the strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is an SCE if and only if $s_i^* \in \arg\max_{s_i\in S_i}\delta_i(s_i)$ for each $i \in N$.*

By contrast, if the game is sufficiently asymmetric such that the maximums of some players' idiosyncratic components are much smaller than others, and τ is small, then the richest players might not take strategies to maximize $\delta_i(s_i)$, as illustrated in the following example.

Example 5. Denote $\Delta_i = \pi_i - \frac{1}{n}\sum_j \pi_j$. Suppose $u_i = \Delta_i$ when $\Delta_i \leq \tau$, and $u_i = -\Delta_i$ when $\Delta_i > \tau$. Suppose $\bar{\delta}_1 > \bar{\delta}_i = \bar{\delta}_L$ for each $i > 1$, and assume that there exists $s_1 \in S_1$ such that $\delta_1(s_1) = \bar{\delta}_L + \tau$. It is easy to verify that the strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ such that $\delta_1(s_1^*) = \bar{\delta}_L + \tau$ while $\delta_i(s_i^*) = \bar{\delta}_L$ for each $i > 1$ constitutes an SCE. In other words, when $\bar{\delta}_1 > \bar{\delta}_L + \tau$, player 1 may choose a different strategy than the one maximizing her idiosyncratic component.

4.3. Heterogeneous players

We now examine what happens if only some players have social comparison preferences. For simplicity, assume that the set N of players is partitioned into two subsets N_1 and N_2 (i.e., $N_1 \neq \emptyset$, $N_1 \subsetneq N$ and $N_2 = N \setminus N_1$), where each player in N_1 has social comparison preferences as above and each player in N_2 is a material payoff maximizer.

Let us focus on symmetric games.[19] Suppose that the material game $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ is symmetric and is decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$. By symmetry, all players have the same set of pure strategies, i.e. $S_i = S_j$ for all $i, j \in N$ and further have the same idiosyncratic component, i.e. $\delta_i(\cdot) = \delta_j(\cdot)$ for all $i, j \in N$. With a slight abuse of notation, let $\hat{S}$ denote the common set of pure strategies. Let $\delta : \hat{S} \mapsto \mathbb{R}$ denote the common idiosyncratic component. Further, we assume that the maximum of $\delta(\cdot)$ exists and can be achieved at s_ℓ^*. Formally, $\delta(s_\ell^*) = \max_{s_\ell\in\hat{S}}\delta(s_\ell)$.

We let $\theta(\Gamma)$ denote *the degree of strategic dependence* of Γ if $\theta = \hat{\theta}/n$, and $\hat{\theta} \in \{1, 2, \ldots, n\}$ is such that:

[19] We can extend the analysis to more general cases with more complex notations.

(i) $\phi(s)$ is a constant for all strategy profiles $s \in \underbrace{\{s^*_\ell\} \times \cdots \times \{s^*_\ell\}}_{\hat{\theta}} \times \underbrace{\hat{S} \times \cdots \times \hat{S}}_{n-\hat{\theta}}$, and

(ii) there exists no smaller $\hat{\theta}$ such that the condition in (i) holds.

For example, for the minimum effort game Γ^{MEG} introduced in Example 2, we have $\theta(\Gamma^{MEG}) = \frac{1}{n}$. For the public goods game Γ^{PGG} introduced in Example 3, we have $\theta(\Gamma^{PGG}) = 1$.

As before, we can define the extended games and social comparison equilibria, where only for the players in N_1 the preferences need to be rewired and are still assumed to be captured by value functions. The following theorem shows that we have the same characterization of social comparison equilibria even if only a small proportion of players compare themselves to others when making decisions, provided that the game's strategic dependence is not too high. The proof is straightforward, and we omit it here.

Theorem 3. *Suppose that the symmetric material game Γ is decomposable with $\{\phi, \{\delta\}_{i\in N}\}$ and that $\theta(\Gamma) \leq \frac{|N_1|}{n}$. Then, the profile $s^* = (s^*_1, \dots, s^*_n)$ is an SCE if and only if $s^*_i \in \arg\max_{s_i \in \hat{S}} \delta(s_i)$ for each $i \in N$, provided that either the condition in Theorem 1 or the condition in Theorem 2 holds.*

The intuition is as follows. When making decisions, the players in N_1 have social comparison preferences, and as analyzed in Theorems 1 and 2, they choose the idiosyncratic optimal strategy s^*_ℓ. The strategic choices of these players make the common component of the material payoff functions stay at a fixed level. This means that the other players, the maximizers of the material payoffs in N_2, have no control over the common component anymore. As a result, these players behave as if they also have social comparison preferences and are motivated to maximize the idiosyncratic components and choose s^*_ℓ.

5. Applications

We now apply our main results to characterize SCE for examples introduced in Section 3. These applications not only illustrate our approach but also demonstrate a social comparison trap: social comparison concerns often result in undesirable outcomes, sometimes producing worse outcomes than any equilibrium of the original material game. Except for Example 2–the minimum effort game, all other examples have a degree of strategic dependence $\theta(\Gamma) = 1$. Thus, to satisfy the condition of Theorem 3, we assume all players have social comparison preferences unless otherwise stated.

Example 1 continued. For the 2 × 2 coordination game, recall that for all players $i \in N$

$\delta_i(A) = 0$ and $\delta_i(B) = d - c$.

It is straightforward that for all $i \in N$,[20]

$$\arg\max_{a\in\{A,B\}} \delta_i(a) = \begin{cases} \{A\} & \text{if } c > d; \\ \{A, B\} & \text{if } c = d; \\ \{B\} & \text{if } c < d. \end{cases}$$

Thus, when the competitive domain in the value function $v(\cdot)$ is sufficiently large ($\tau \geq 2|c - d|$), we can apply Theorem 1 and obtain the unique SCE such that all players choose strategy A when $c > d$ (resp. B when $c < d$). When $\tau > 0$ is small, Theorem 2 applies whenever $|c - d| \leq \tau$ so that the richness condition holds. This game has a degree of strategic dependence $\theta(\Gamma) = 1$. However, due to the nature of coordination, when one player has social comparison preferences, the SCE outcome is obtained.

[20] For the study of general two-person matrix games, see Section 6.5.

Given that $b > a$, $a > d$, and $a > c$, we have $b > a > \max\{c, d\}$. Example 1 shows that players will choose a maxmin strategy in the material 2 × 2 coordination game which maximizes their worst-case material payoff.[21] The result in Example 1 is related to Sawa and Wu (2018a,b) and Nax and Newton (2019) where agents in a finite population are randomly paired to play a 2 × 2 coordination game and regard the average material payoffs in the population as the reference point. When $d > c$, it holds that $b + d > a + c$, which implies that the maxmin strategy B is risk dominant in the sense of Harsanyi and Selten (1988). In the terminology of Sawa and Wu (2018a,b), strategy B is loss-dominant.

Example 2 continued. In the minimum effort game, the idiosyncratic component is $\delta_i(e_i) = -\delta e_i$ for all $i \in N$, where $0 < \delta < 1$. Thus, $\arg\max_{e_i\in E}(-\delta e_i) = \{e_1\}$, provided that $0 < e^1 < \cdots < e^\rho < +\infty$. Therefore, according to Theorem 1, when the competitive domain is broad enough ($\tau \geq 2\delta e^\rho$), all players choose the lowest effort e_1 in the unique SCE.[22] Or, according to Theorem 2, if effort levels are sufficiently fine-grained, i.e., $e^{k+1} - e^k \leq \tau/\delta$ for each $k \in \{1, 2, \dots, \rho - 1\}$, so that the richness condition holds, then the same outcome follows. This game has a degree of strategic dependence $\theta(\Gamma^{MEG}) = 1/n$. Thus, according to Theorem 3, we obtain this SCE as long as at least one player has social comparison preferences.

Example 2 shows that social comparison preferences can dramatically change equilibrium in the minimum effort game compared to the standard analysis of material payoff maximizers. In the original material game, any strategy profile where all players choose the same effort level constitutes a strict Nash equilibrium. Thus, multiple strict Nash equilibria arise. However, once we incorporate social comparison concerns, every player has a strictly dominant strategy: choosing the lowest effort.

Moreover, suppose the material payoff function has significantly changed, altering best-response strategies in the original game. However, it can happen that this change only affects the common component $\phi(s)$, while leaving idiosyncratic components unchanged. Consequently, the change may not alter best response strategies of competitive players who aim to maximize their relative standing. To illustrate, consider the minimum effort game. Suppose that players' gains from the project depend not on the minimum but on the *maximum* effort among all players. This change would alter best response strategies of material payoff maximizing players. However, the idiosyncratic component remains $\delta_i(e_i) = -\delta e_i$, resulting in the same SCE.

Example 3 continued. In the public goods game, the idiosyncratic component is $\delta_i(e_i) = -ce_i$, where $c > 0$. Thus, $\arg\max_{e_i\in[0,+\infty)}(-ce_i) = \{0\}$. We cannot apply Theorem 1 to analyze this game for any finite τ because the idiosyncratic component would be unbounded in the case of $v'(\Delta_i) < 0$ when $\Delta_i > \tau$. However, we can apply Theorem 2 to obtain the unique SCE in which all players contribute nothing, even when $\tau > 0$ is arbitrarily small. This is because Theorem 2 holds for all symmetric decomposable games with convex strategy spaces and continuous idiosyncratic components. This game has a degree of strategic dependence $\theta(\Gamma^{PGG}) = 1$, thus requiring all players to have social comparison preferences to achieve the contributing nothing outcome.

In the public goods game in Example 3, when all other players make zero contribution, a material payoff maximizing player chooses

[21] See Section 6.6 for further discussion on the relationship between maxmin strategies and SCE.

[22] When players have inequality-averse preferences as in Fehr and Schmidt (1999) and Bolton and Ockenfels (1999), it can be shown that the profile $(e^\rho, \dots, e^\rho)$ is an equilibrium. Section 6.4 provides further discussion.

a strictly positive contribution level e^*, where $b'(e^*) = c$.[23] However, if players have social comparison preferences, they would contribute nothing, regardless of other players' contributions. This is because every player cares only about maximizing their idiosyncratic components, which involve only the cost part of their contribution. In fact, since positive contributions benefit all players to the same degree while the contributor alone bears the costs, positive contributions reduce the contributor's social status.

Example 4 continued. In the tragedy of the commons game, the idiosyncratic component is $\delta_i(k_i) = \ln(1+k_i)$. Thus, $\arg\max_{k_i \in [0,K]} \ln(1+k_i) = \{K\}$. When the competitive domain is large (i.e., when $\tau \geq 2\ln(1+K)$), we can apply Theorem 1 and obtain the unique SCE in which every player consumes the largest possible level of the resource. When $\tau > 0$ is small, we can apply Theorem 2 to obtain the same outcome, because this game is symmetric with a convex strategy space and continuous idiosyncratic components. This game has a degree of strategic dependence $\theta(\Gamma^{TCG}) = 1$, thus requiring all players to have social comparison preferences to achieve this outcome. Note that in the original material game, each player consumes the level $nK/(n+1)$ in the unique Nash equilibrium, where $nK/(n+1) < K$. Thus, by playing the unique SCE, all players overextract to the maximum and become strictly worse off.

The examples in this section demonstrate a social comparison trap: In the unique SCE in each of these examples, players are worse off than they would be if they simply aimed to maximize their material payoffs. The minimum effort game in Example 2 and the public goods game in Example 3 present situations with positive externalities. In both games, choosing a higher effort than the lowest level can sometimes benefit the effort taker herself. Thus, although under positive externalities players generally exert less effort in a Nash equilibrium than the socially optimal level, they do often exert a strictly positive effort for their own benefits. However, under social comparison preferences, players will forgo these benefits to themselves, because exerting a positive effort benefits other players even more, thus reducing their own relative standing. Thus, social comparison pushes every player to choose the lowest effort in the minimum effort game and zero contribution in the public goods game. By contrast, the tragedy of the commons game in Example 4 presents a situation with negative externalities. However, it leads to a similar pessimistic outcome: although overextracting resources harms the players themselves, it harms all other players even more, thus raising their own relative standing. Therefore, under social comparison preferences, all players end up playing the unique SCE of overextracting to the maximum, making everyone worse off.

6. Extensions and concluding remarks

6.1. Alternative social reference points

We have so far assumed that all players take the population average of material payoffs as their reference points. We now examine whether our results hold for alternative reference points. First, as people may look upward in social comparisons (Blanchflower and Oswald, 2004; Immorlica et al., 2017), we consider the case where the reference point is the maximum material payoff among all players.[24] Formally, for each $s \in S$, players use the maximum of the material payoff profile, $r_i^{max}(s) = \max_{1\leq j\leq n} \pi_j(s)$, as their reference point rather than the average. Assume $\delta_i(s_i) \neq \delta_i(s_i')$ if $s_i \neq s_i'$. As in Section 4, the difference $\Delta_i(s_i, s_{-i}) = \pi_i(s_i, s_{-i}) - r_i^{max}(s_i, s_{-i})$ is then

$$\Delta_i(s_i, s_{-i}) = \delta_i(s_i) - \max_{1\leq j\leq n} \delta_j(s_j),$$

which depends only on the idiosyncratic components. We observe that when a player already chooses a strategy s_i such that $\delta_i(s_i) = \max_{1\leq j\leq n} \delta_j(s_j)$, she would not want to revise her strategy downward. However, she is indifferent with any s_i' such that $\delta_i(s_i') \geq \delta_i(s_i)$. Therefore, unlike Theorems 1 through 3, if the profile $s^* = (s_1^*, s_2^*, \dots, s_n^*)$ induces the same value of idiosyncratic components $\delta_i(s_i^*)$ for all players, then it constitutes an equilibrium, even if some players' strategies do not maximize their idiosyncratic components. However, only when all players' strategies maximize their idiosyncratic components does it constitute a *strict* equilibrium, as only in this case each player's best-response strategy is unique.

Second, our analysis extends to other order statistics as the reference point, such as the second-highest material payoff or the minimum payoff among all players. In either case, the players' best-response strategies remain the maximizers of their idiosyncratic components. Thus, the same reasoning from Theorems 1 through 3 applies.

Third, our analysis extends to heterogeneous reference points across players. For instance, some players might compare themselves to the population average, while others compare with the second-highest payoff. As long as the comparison outcome $\Delta_i(s_i, s_{-i})$ depends only on the idiosyncratic components and the maximizer of $\Delta_i(s_i, s_{-i})$ remains the same as the maximizer of $\delta_i(s_i)$, our analysis and main results apply.

6.2. Social comparison within networks

Studies indicate that social comparison can be local, occurring among friends, neighbors, or colleagues within an individual's social network (Immorlica et al., 2017; Ghiglino and Goyal, 2010; Cheng and Xing, 2022). Our model can be extended to include this aspect, and all our results remain valid. Let $N_i \subset N$ denote the set of i's network neighbors, where $N_i \neq \emptyset$ and $i \notin N_i$, and let $d_i = |N_i|$. Let $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ summarize all material consequences. When doing social comparison, players now compare with only those within their neighborhood:

$$u_i(s) = v\Big(\pi_i(s) - r_i^{local}(s)\Big), \quad \text{where} \quad r_i^{local}(s) = \frac{1}{d_i+1}\sum_{j\in N_i\cup\{i\}} \pi_j(s).$$

As in our main analysis, the difference between a player's own payoffs and the reference point depends only on the idiosyncratic components:

$$\Delta_i^{local}(s) = \pi_i(s) - r_i^{local}(s) = \frac{d_i}{d_i+1}\delta_i(s_i) - \frac{1}{d_i+1}\sum_{j\in N_i} \delta_j(s_j).$$

Thus, $\arg\max_{s_i\in S_i} \Delta_i^{local}(s_i, s_{-i}) = \arg\max_{s_i\in S_i} \delta_i(s_i)$. Consequently, by the same arguments, we can derive results similar to Theorems 1 through 3.[25]

6.3. Social comparison in non-decomposable games

A social comparison equilibrium may also exist in non-decomposable games. One example is the Cournot oligopoly game. Consider $n \geq 3$ firms that produce a homogeneous good and compete in quantities $q_i \geq 0$. Let $D(Q)$ denote the inverse demand (i.e., the market price) given total output $Q = \sum_i q_i$, with $D'(Q) < 0$ for all $Q \in [0, Q_{max}]$, $D(0) > 0$, and $D(Q) = 0$ for all $Q \geq Q_{max}$. Let $C(q_i)$ denote the cost of producing q_i, with $C'(q_i) > 0$. The payoff function is then

$$\pi_i(q_i, q_{-i}) = D\Big(\sum_j q_j\Big) q_i - C(q_i).$$

Applying Proposition 2, we can show that this game is not decomposable. However, this game is an exact potential game when $D(\cdot)$ is linear, since in the linear case we have $\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} = D'(Q)$ for each

[23] As shown by Bramoullé and Kranton (2007), there also exists another class of Nash equilibria where every player contributes a level e^*/n of public goods.

[24] We are very grateful to the referee for pointing out this possibility.

[25] In fact, the condition for Theorem 1 can be weakened to $\tau \geq |\delta_i(s_i)| + |\delta_j(s_j)|$ for each $s_i \in S_i$, $s_j \in S_j$, $i \in N$, and $j \in N_i$. Similarly, the condition for Theorem 2 can be weakened to $|\bar{\delta}_i - \bar{\delta}_j| \leq \tau$ for each $i \in N$, $j \in N_i$.

$i, j \in N$. Thus, this provides an example of a potential game that is not decomposable when there are more than two players.

We next show that the Walrasian (perfect competition) equilibrium is the only social comparison equilibrium of this game. The Walrasian quantity q^W is defined by $D(nq^W)q^W - C(q^W) > D(nq^W)q - C(q)$ for all $q \neq q^W$. The Walrasian quantity exhibits a property called *spite*: $D((n-m)q+mq^W)q^W - C(q^W) > D((n-m)q+mq^W)q - C(q)$ for all $q \neq q^W$ and $1 \leq m < n$ (Schaffer, 1989). This property implies that: (1) although deviating from q^W may increase the profits of the deviators, the profits of the firms that produce q^W will increase more; (2) switching to q^W from a different monomorphic profile, e.g., the Nash equilibrium, may reduce the profits of the deviators, with the profits of other firms being reduced even more. Thus, firms that pursue higher payoffs relative to others will choose the Walrasian quantity. This implies that the Walrasian equilibrium is the only social comparison equilibrium. Since the Walrasian quantity is larger than the Nash quantity, social comparison preferences lead to more competitive and more negative outcomes for every firm.

6.4. Trade-off between social comparison and payoff maximization

As in most social preference models, what if players care about their absolute material payoffs alongside their relative standing? We now examine this possibility. Assume a linear value function to better connect with the literature (e.g. Fehr and Schmidt, 1999; Charness and Rabin, 2002). Specifically, consider the utility function

$$u_i(s) = \lambda \pi_i(s) + (1-\lambda)\Delta_i(s),$$

where $0 \leq \lambda \leq 1$ and $\Delta_i(s) = \pi_i(s) - \frac{1}{n}\sum_j \pi_j(s)$. When $\lambda = 0$, this utility function represents the sole competitive preference we have examined. When $0 < \lambda < 1$, players care about both their material payoffs in absolute terms and their relative standing. In this framework, while equilibria may differ from the SCEs characterized in our main analysis, social comparison still influences choices and outcomes in the direction that SCE underscores. In particular, the SCEs we characterize represent the limit outcomes when λ is small. For instance, in Example 2, the minimum effort game, all players choosing the lowest effort is the unique equilibrium when $\lambda < \frac{n-1}{n-\delta}\delta$. By contrast, when $\lambda > \frac{n-1}{n-\delta}\delta$, multiple equilibria arise and are the same as those in the material game. In Example 3, the public goods game, the first-order condition is

$$\frac{\partial u_i}{\partial e_i} = \lambda\left[b'\left(\sum_j e_j\right) - c\right] - (1-\lambda)\left(1 - \frac{1}{n}\right)c = 0 \tag{13}$$

for the case of $e_i > 0$. The second term above represents marginal losses in relative standing from contributing more. When $\lambda = 1$, we obtain the standard symmetric Nash equilibrium of the material game: $e^{NE} = \frac{1}{n}b'^{-1}(c)$.[26] When $0 < \lambda < 1$, we obtain a *generalized SCE*: $e^*(\lambda) = \max\{0, \frac{1}{n}b'^{-1}(c + (\frac{1}{\lambda} - 1)(1 - \frac{1}{n})c)\}$. Observe that $e^*(\lambda)$ increases with λ, has limit e^{NE} as λ approaches 1, and $e^*(\lambda) = e^{SCE} = 0$ when $\lambda > 0$ is sufficiently small. These two examples illustrate how equilibrium is shaped by the trade-off between payoff maximization and social comparison concerns.

6.5. Social comparison equilibrium and Nash equilibrium

Example 4 shows that an SCE may not be an NE. This is not surprising because in an SCE players with competitive preferences maximize their relative payoffs, while in an NE players maximize their absolute payoffs. Thus, roughly speaking, a strategy profile would be both an NE and an SCE if and only if given other players' strategies, no player can improve her absolute payoff or her relative payoff by deviating to another strategy. In other words, there is no conflict between maximizing $\pi_i(s_i, s_{-i})$ and maximizing $\Delta_i(s_i, s_{-i})$. This condition is satisfied in constant-sum games, in which a higher absolute payoff also implies a higher relative payoff. Therefore, NE coincides with SCE in constant-sum games. In addition, the two equilibrium concepts coincide in perfectly competitive markets in which each individual is atomic so that they cannot influence the payoff of others, but the two concepts differ in oligopoly markets in which players have market power (see Section 6.3 for analysis of the Cournot oligopoly game).

To better understand the precise relationship between (pure strategy) SCE and NE, we derive SCE conditions in the context of two-person matrix games for players with competitive preferences (i.e., τ is sufficiently large). First, we consider an $n \times n$ symmetric game with (material) payoff matrix $(a_{ij})_{n\times n}$. Without loss of generality, assume $a_{ij} = a_{kl}$ if and only if $i = k$ and $j = l$. The game does not have any asymmetric SCE, because the player with a low material payoff can increase his relative income to zero by imitating his opponent's strategy. In addition, a symmetric strategy pair (s_i, s_i) is a strict SCE if and only if $a_{ij} > a_{ji}$ for all $j \neq i$, i.e., deviating to another strategy will cause the relative income to become negative. The above condition implies that the game has at most one strict SCE. That is, if (s_i, s_i) is a strict SCE, then all other pairs of symmetric strategies are not strict SCE. To see this, suppose that the two players use the strategy s_j with $j \neq i$. In this case, a player can obtain a positive relative payoff by deviating to s_i since $a_{ij} > a_{ji}$. In contrast, (s_i, s_i) is a strict NE if and only if $a_{ii} > a_{ji}$ for all $j \neq i$. Therefore, an NE coincides with an SCE only when $a_{ii} > a_{ji}$ and $a_{ij} > a_{ji}$ for all $j \neq i$.

Next, we consider an $m \times n$ asymmetric game with payoff matrix $(a_{ij}, b_{ij})_{m\times n}$, and assume $a_{ij} - b_{ij} = a_{kl} - b_{kl}$ if and only if $i = k$ and $j = l$. In this game, a strict SCE can be asymmetric. A strategy pair (s_i, s_j) is a strict SCE if and only if $a_{il} - b_{il} > a_{ij} - b_{ij} > a_{kj} - b_{kj}$ for all $k \neq i$ and/or $l \neq j$, where the first inequality means that the second player cannot improve his relative payoff by deviating to s_l and the second inequality means that the first player cannot improve his relative payoff by deviating to s_k. We note that this condition clearly differs from the NE conditions, which are $a_{ij} > a_{kj}$ for all $k \neq i$ and $b_{ij} > b_{il}$ for all $l \neq j$. In general, an asymmetric game also has at most one strict SCE. In fact, if there is another strict SCE, denoted by (s_k, s_l) with $k \neq i$ and/or $l \neq j$, then this requires $a_{kj} - b_{kj} > a_{kl} - b_{kl} > a_{il} - b_{il}$, which contradicts $a_{il} - b_{il} > a_{kj} - b_{kj}$.

6.6. Social comparison equilibrium and maxmin strategies

Next, we examine whether players with social comparison preferences will choose maxmin strategies in decomposable games. A strategy $s_i \in S_i$ is maxmin if $s_i \in \arg\max_{s'_i \in S_i} \min_{s'_{-i} \in S_{-i}} \pi_i(s'_i, s'_{-i})$. Consider a game $\Gamma = (N, \{S_i\}_{i\in N}, \{\pi_i\}_{i\in N})$ decomposable with $\{\phi, \{\delta_i\}_{i\in N}\}$. Given decomposability, it holds that for each $s_i \in S_i$,

$$\arg\min_{s_{-i}\in S_{-i}} \phi(s_i, s_{-i}) = \arg\min_{s_{-i}\in S_{-i}} (\phi(s_i, s_{-i}) + \delta_i(s_i)) = \arg\min_{s_{-i}\in S_{-i}} \pi_i(s_i, s_{-i}).$$

Denote $\hat{\phi}_i(s_i) \equiv \min_{s_{-i}} \phi(s_i, s_{-i})$. Then, each player i's maxmin strategies are those $s_i \in S_i$ that maximize $\hat{\phi}_i(s_i) + \delta_i(s_i)$. Therefore, if $\hat{\phi}_i(s_i)$ and $\delta_i(s_i)$ are monotone in the same direction, then $\arg\max_{s_i\in S_i} \delta_i(s_i) = \arg\max_{s_i\in S_i} \hat{\phi}_i(s_i) + \delta_i(s_i)$, so that a social comparison player will choose maxmin strategies. For instance, in Example 2, the minimum effort game, we have $\hat{\phi}_i(e_i) = e_1$ and $\delta_i(e_i) = -\delta e_i$; both decrease (weakly) with e_i. Thus, the unique SCE effort level e_1 that maximizes $\delta_i(e_i)$ is also the maxmin strategy.

By contrast, if $\hat{\phi}_i(s_i)$ and $\delta_i(s_i)$ are not monotone in s_i in the same direction, then maximizers of $\delta_i(s_i)$ may differ from those of $\hat{\phi}_i(s_i) + \delta_i(s_i)$, so that SCE differs from maxmin strategies. For example, in Example 3, the public goods game, we have $\hat{\phi}_i(e_i) = b(e_i)$ while $\delta_i(e_i) = -ce_i$. Thus, the maxmin strategy is e^* that satisfies $b'(e^*) = c$, which generally differs from $e^{SCE} = 0$. The reason is the following.

[26] There also exist a collection of asymmetric Nash equilibria in which the sufficient public goods amount $b'^{-1}(c)$ is shared by a subset of rather than all players. For instance, player 1 alone contributes $e_1 = b'^{-1}(c)$ while all others free ride with zero contribution.

In a decomposable game, the material payoff function for each player consists of two parts: the common component and their idiosyncratic component. While the maxmin strategies in the material game are determined by both parts, the best-response strategies in the extended game are determined only by the idiosyncratic component. As a result, players with social comparison preferences may not choose maxmin strategies in the material games. The following statement summarizes this observation.

Proposition 5. *A SCE strategy may not be a maxmin strategy.*

6.7. Evolutionary explanation for social comparison equilibrium

Although an SCE need not be an NE of the original game, it can be the outcome of stochastic learning dynamics or evolutionary processes. For example, in the Cournot oligopoly game in Section 6.3, if firms imitate the most successful quantity they observed, the Walrasian equilibrium is uniquely selected in the long run (Vega-Redondo, 1997). Moreover, a player with social comparison preferences may take spiteful actions that harm both themselves and others if these actions lead to a relative-payoff advantage. A typical example of spiteful behavior is costly punishment, i.e., paying a cost to reduce another individual's payoffs. Costly punishment has been widely observed in human societies (Henrich et al., 2006). Previous studies based on evolutionary game theory suggest that such spiteful behavior could be evolutionarily stable if the punishment-to-cost ratio is high and the population is not sufficiently large (Hamilton, 1970; Rand and Nowak, 2011). Similar mechanisms could provide an evolutionary explanation for social comparison preferences.

To conclude, this paper provides a framework for understanding the implications of social comparison preferences and the social comparison trap. It demonstrates that individuals often ignore common payoff component to maximize the idiosyncratic component, thereby improving their relative standing at the expense of overall group welfare and their own absolute payoffs. Since these preferences may be persistent or even evolutionarily rooted, the key policy implication is that simply highlighting collective gains is likely ineffective. Instead, effective policies must account for these powerful status-seeking motives, potentially by structuring incentives so that personal status aligns with, rather than conflicts with, socially desirable outcomes.

CRediT authorship contribution statement

Zhiwei Cui: Writing – review & editing, Writing – original draft, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Xueheng Li:** Writing – review & editing, Writing – original draft, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Boyu Zhang:** Writing – review & editing, Writing – original draft, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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